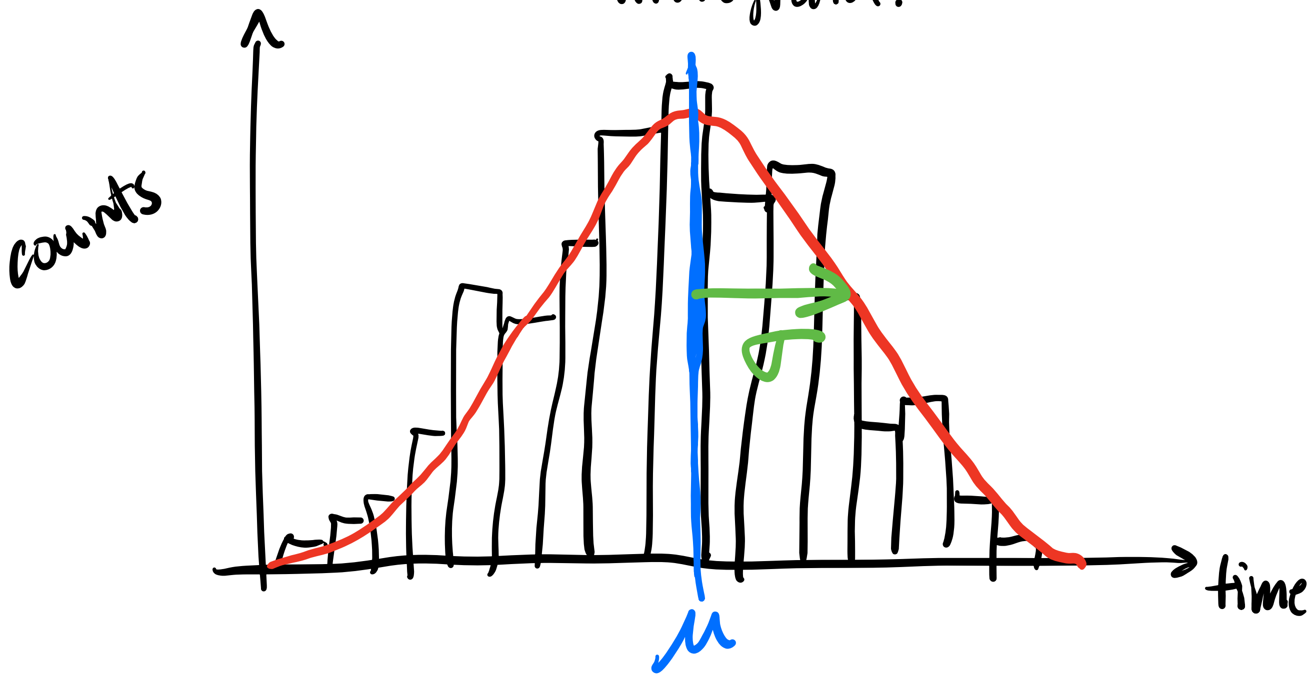
PHYS 231 - Sept. 18, 2023

<u>Last Time</u> Plot results of a repeated measurement as a histogram.



$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$$ mean.

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu)^2}$$ standard deviation

Gaussian Dist'n

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\left(\text{when } x = \mu \pm \sigma \quad f(x) \Rightarrow \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}} = \frac{.607}{\sigma\sqrt{2\pi}} \right)$$

If we meas. a quantity N times using same method, should report μ as our best estimate of the true value.

For the uncertainty in μ we should report ...

Expect our estimate of the mean μ to improve as we increase the number of trials N.
However, we have observed that the std dev. σ does not change as we increase the no. of trials N.

Therefore, σ on its own is <u>not</u> a good est. of the uncertainty in μ.

The proper est. of the uncertainty in μ is given by the standard error σ_μ.

$$\boxed{\sigma_\mu = \frac{\sigma}{\sqrt{N}}}$$

Suppose we do $\underbrace{M}_{1000}$ sets of $\underbrace{N}_{100}$ measurements

Calc. the mean of each set of 100 (N) meas.
$\Rightarrow$ 1000 (M) value of μ.

When we plot the dist'n of means, it has a width of $\sigma_\mu = \frac{\sigma}{\sqrt{N}}$ which is the standard error.

σ_μ has the desired property that it decreases as N increases.

When you repeat many trials of an experiment, the final result that you report should be:

$$\boxed{x = \mu \pm \frac{\sigma}{\sqrt{N}}}$$

If we meas. $x_1 \pm \Delta x_1$ and $x_2 \pm \Delta x_2$, then what is the uncertainty in

$$y = x_1 + x_2 ?$$

i.e. what is Δy?

The uncertainty in y is actually given by

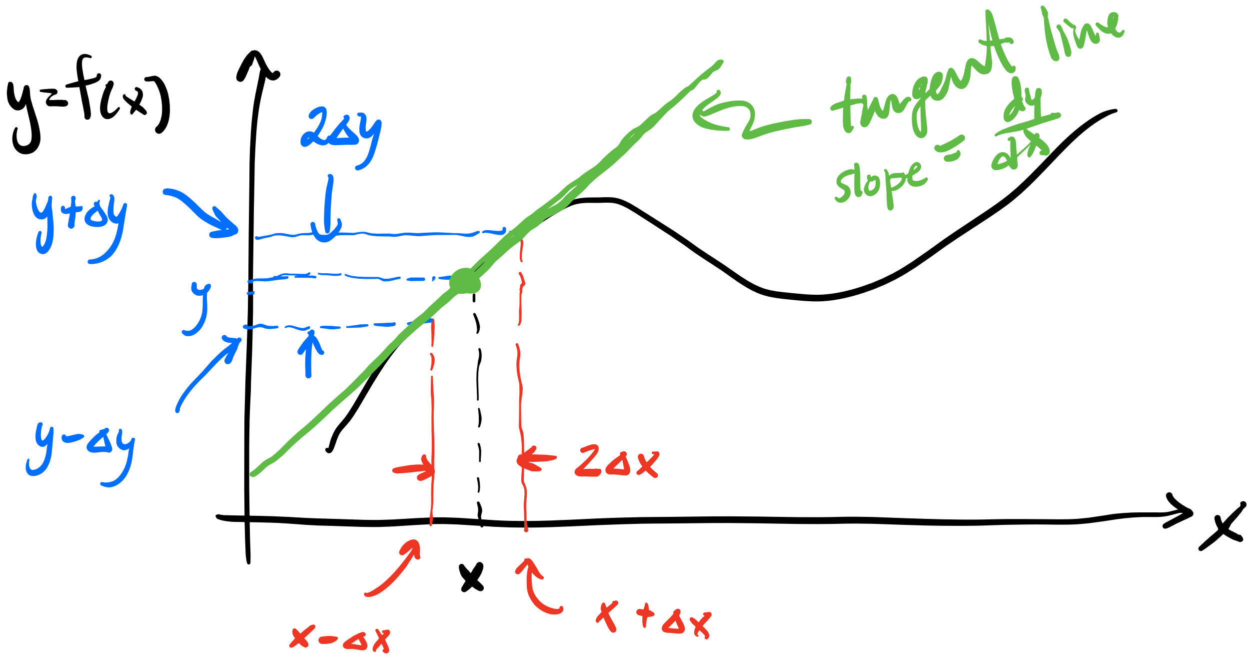
$$\Delta y = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2}$$

This is called the quadrature sum of Δx_1 & Δx_2.

$$\sqrt{(\Delta x_1)^2 + (\Delta x_2)^2} < \Delta x_1 + \Delta x_2$$

How do we calc. uncertainty in $y = f(x)$ when we know $x \pm \Delta x$?

i.e. How do we find Δy?



Slope of tangent $\frac{dy}{dx} = \frac{\text{rise}}{\text{run}} = \frac{\cancel{2}\Delta y}{\cancel{2}\Delta x}$

$$\therefore \boxed{\Delta y = \left|\frac{dy}{dx}\right| \Delta x}$$

If we know $x \pm \Delta x$ & then calc $y = f(x)$, then uncertainty in

y is given by $\Delta y = \left|\frac{dy}{dx}\right| \Delta x$.

Now suppose we meas. $u \pm \Delta u$ and $v \pm \Delta v$ and we want to calc. $y = f(u,v)$.

What is Δy?

First, we find the uncertainty in y due to the u variable.

$$\Delta y_u = \left| \frac{\partial f(u,v)}{\partial u} \right| \Delta u$$

Then find the contribution due to v variable

$$\Delta y_v = \left| \frac{\partial f(u,v)}{\partial v} \right| \Delta v$$

Finally, we combine the two contributions using a quadrature sum to find the net uncertainty

$$\Delta y = \sqrt{(\Delta y_u)^2 + (\Delta y_v)^2}$$

$$= \sqrt{\left(\frac{\partial f(u,v)}{\partial u}\Delta u\right)^2 + \left(\frac{\partial f(u,v)}{\partial v}\Delta v\right)^2}$$

Propagation of Errors

In general, for a fcn of any no. of variables

$$y = f(x_1, x_2, \ldots, x_N)$$

$$\Delta y = \sqrt{\left(\frac{\partial f}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\Delta x_2\right)^2 + \ldots + \left(\frac{\partial f}{\partial x_N}\Delta x_N\right)^2}$$